Int. J. Heat Mass Transfer. Vol. 15, pp. 279–299. Pergamon Press 1972. Printed in Great Britain

FLUID FLOW AND HEAT TRANSFER WITH TWO CYLINDERS IN CROSS FLOW

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(*Received* 18 *December* 1970 *and in revised form* 13 *May* 1971)

Abstract—Experimental investigations of the flow and heat transfer around two cylinders in cross flow (one of them in the wake of the other) has been carried out. Distributions of the static pressure coefficients and Nusselt number on the surfaces of both cylinders have been determined by measurements. Our experimental results and knowledge of the flow processes around the single cylinder enables us to form an idea of the mechanism and pattern of the flow between and around two cylinders, and also to explain the mutual influence of the cylinders and the characteristic changes of the form drag and mean heat transfer coefficient.

Reynolds number was varied in the range $1{\cdot}2 \,.\, 10^4 \leqslant Re \leqslant 4 \,.\, 10^4$ and the distance between the cylinders in the interval $1{\cdot}6 \leqslant L/D \leqslant 9$.

Two different characteristic flow patterns was found, depending on whether the distance between the cylinders is greater or smaller than $3{\cdot}8\ D$. For $L < 3{\cdot}8\ D$ the closed vortex region between the cylinders is formed.

Mean Nusselt number for the 1st cylinder is proportional to $Re^{0{\cdot}6}$, as for the single cylinder for all distances.

Mean Nusselt number for the 2nd cylinder is proportional to $Re^{0{\cdot}6}$ for $L > 2{\cdot}7\ D$, and to $Re^{\frac{3}{4}}$ for $L \leqslant 2{\cdot}7\ D$.

NOMENCLATURE

C_D, total drag of two cylinders;

C_{Dp}, $\frac{1}{\pi}\int_0^{\pi} C_p^0 \,.\, \cos\phi \,.\, \mathrm{d}\phi$, form drag coefficient;

C_p^0, $(p_\phi - p_{\max})/0{\cdot}5\,\rho U_\infty^2$, local static pressure coefficient;

D, cylinder diameter;

H, height of the experimental section;

K, $Nu.Pr^{-0{\cdot}4}$

L, distance between the axes of the cylinders;

Ma, Mach number;

Nu, $\alpha_\phi D/\lambda_f$, Nusselt number;

Nu_m, mean Nusselt number defined on the mean integral temperature difference;

Nu_A, maximum Nusselt number near the reattachment point on the 2nd cylinder;

Pr, $c_p \,.\, \mu/\lambda$, Prandtl number;

Re, $U_\infty \,.\, D/\nu$, Reynolds number;

S_f, $l_f \,.\, f/U_\infty$, Strouhal number;

SL, laminar boundary-layer separation point or transition point;

ST, turbulent boundary-layer separation point;

U, local mean velocity in the experimental section;

U_∞, mean velocity upstream the cylinders at the channel axes;

c_p, specific heat;

f, frequency of shedding in the cylinder wake;

l_f, vortex formation region length;

Nu_{sr}, $(1/\pi)\int_0^{\pi} Nu \,.\, \mathrm{d}\phi$, mean integral Nusselt number;

p_ϕ, local static pressure on the cylinder surface at angle ϕ;

$p_{\max}$, maximal local static pressure;

q, heat flux;

t, temperature;
u', longitudinal velocity fluctuation;
x, y, coordinates measured from cylinder axes;
α_ϕ, $q_w/(t_w - t_f)$ heat transfer coefficient;
δ, thickness of the cylinder wall;
λ, heat conductivity;
Λ, macroscale of turbulence;
μ, $\rho \cdot \nu$, dynamic viscosity;
ρ, density;
ϕ, central angle measured from front stagnation point;
ϕ^-, central angle measured from reattachment point;
ϕ^+, central angle measured from point of maximum Nusselt number on the second cylinder.

Subscripts
f, value far from the wall;
w, value at the wall.

1. INTRODUCTION

INVESTIGATION of the flow and heat transfer in tube banks in cross flow is still of great practical interest due to its wide use in the engineering practice and also to insufficiently investigated effects of particular geometrical parameters on the flow processes [1].

Simple experimental models (two cylinders in cross flow, one of them being in the wake of the other) simulate fairly well the conditions of the flow processes in an in-line tube bank. Conclusions drawn from such experiments can be successfully used for the analysis and evaluation of the data obtained for tube banks. Such experiments also provide useful information on heat transfer in flows with separated regions.

Data found in the literature on the flow around a cylinder in the wake of some preceding body (with exception of the cylinder in a bank) are scarce.* According to the literature available to us, J. R. Pannell *et al.* were the first to make experiments in this field [2], but they measured only the total drag of two cylinders at various mutual distances.

* No published results on the heat transfer have been found.

A.F. Charwat *et al.* [3] investigated the case of a cylinder in the wake of a wedge in supersonic flow. Distribution of the static pressure on the cylinder surface was measured, and the distance at which a closed vortex region (free cavity) is formed between the cylinder and the wedge, was determined.

J. H. Gerrard [4] investigated the shedding frequency in the wake of a cylinder with a splitter plate normal to the wake axis. He noticed a sudden change in shedding frequency at a certain distance between the plate and a cylinder.

It is not possible to get a clear idea about the flow processes around and between two cylinders on the basis of the such scarce experimental information.

2. ON THE PROBLEM

In explaining the experimental results the present authors depend to a great extent on the well known results of the investigations of the flow around a single cylinder [4–9]. This part of the paper deals with those characteristics of the flow around a single cylinder which are, according to the present authors, essential for understanding the processes controlling the flow and heat transfer from two cylinders.

Over a wide Reynolds number range ($3 \cdot 10^2 \leqslant Re \leqslant 2\text{–}5 \cdot 10^5$) the boundary layer at the front side of the cylinder is laminar and separates at about 80°. After separation, the boundary layer behaves as a free shear layer and its inner edge, otherwise very unstable, soon rolls up into one or several vortices. These vortices are formed in turn and shed downstream [4, 5, 7, 8]. The vortex formation region is schematically presented in the Fig. 1. The length of the vortex formation region in the Reynolds number range $10^3 \leqslant Re \leqslant 10^4$ is constant (about 2·5 D), and for higher Reynolds numbers decreases [5] (Fig. 2, open points). In this, subcritical, regime of flow around a single cylinder, transition to

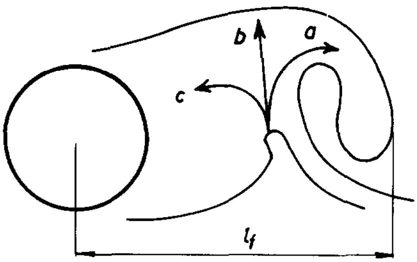

FIG. 1. Mechanism of vortex formation.

turbulence occurs after laminar boundary-layer separation, but within the vortex formation region. With the increase of Reynolds number, the position of transition to turbulence moves towards the cylinder [5] (Fig. 2, full points). When the critical Reynolds number is reached ($Re = 2\text{–}5 \cdot 10^5$) the transition occurs before the separation point. Because of the presence of the turbulent boundary layer the separation point is shifted downstream (at about 135°), resulting in a significant narrowing of the turbulent wake, decrease of the form drag coefficient and increase of shedding frequency (supercritical regime).

When the two cylinders are in cross flow it might be assumed that, at large distances (when the closed vortex region is not yet formed) the

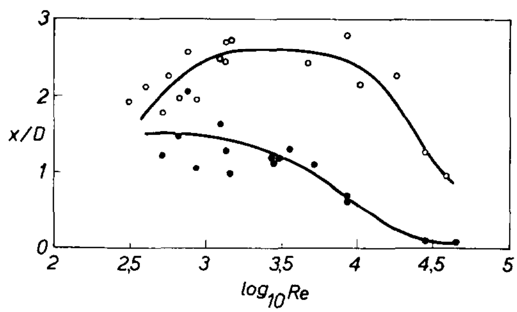

FIG. 2. Length of vortex formation region and region of laminar flow in the wake of single cylinder (measured from cylinder center).

○ vortex formation region
● laminar flow region

presence of the 2nd cylinder* will not affect the processes of the flow and heat transfer developing on the 1st. Namely, these processes will be analogous to those on the single cylinder in infinite fluid flow (that is for our experiment—in subcritical flow regime). At smaller distances when a closed vortex region (or free cavity) is formed between cylinders, the influence of the 2nd cylinder on the process developing on the 1st will be transferred through the vortex. This influence will probably be weak, so that the flow around the 1st cylinder should not be essentially changed.

As the 2nd cylinder is located in the wake of the 1st, the later will affect the processes developing on the former, at all mutual distances: at large distances due to the increased turbulence level [10], at smaller distances due to the combined effect of the turbulence level and the nonuniformity of the oncoming mean flow and at last due to the formation of the closed vortex region.

The turbulence of the main flow affects the mean heat-transfer coefficient for a single cylinder in an infinite fluid flow in the following ways: by increasing the heat transfer in the laminar boundary layer, by stimulating transition of the laminar boundary layer into a turbulent one (with higher heat-transfer coefficients), by shifting the separation point and by changing the flow characteristics in the wake region [10].

Experimental results of the effect of turbulence intensity on the heat transfer around the front stagnation point for different main stream turbulence intensities show quantitative discrepancies with each other [10]. In these experiments only the turbulence intensity was controlled, and no account was taken of the macroscale of turbulence, which, however, according to the analysis of J. Kestin [10] is essential for the control of the similarity of turbulent fields.

* The 1st is the cylinder in the undisturbed fluid flow, the 2nd the cylinder in the wake of the 1st.

Van der Hegge Zijnen [11] found that turbulence of the main stream has the largest influence if the macroscale is of the order of cylinder diameter. A significant increase of the heat transfer on the cylinder in the wake of preceding one, should be expected due to increased turbulence intensity and especially because the macroscale of the oncoming flow is of the order of the cylinder diameter.

In a number of investigations in the vortex regions (separated flow with recirculation) it has been found that the mean Nusselt number is proportional to $Re^{\frac{2}{3}}$. P. D. Richardson [12] has found that this is valid in the vortex regions behind bluff bodies. H. R. Knight [13] has obtained the same for the vortex region formed by a sudden tube expansion. The Nusselt number at the reattachment point is also proportional to $Re^{\frac{2}{3}}$. Therefore, this dependence should also be expected for the vortex region between two cylinders in cross flow.

3. EXPERIMENTAL APPARATUS

Two smooth cylinders (Fig. 3) of circular cross section ($D = 100$ mm) were placed in the experimental section (cross section 500×500 mm^2) of an open wind tunnel. The distance between the cylinders could be varied in the range of $1{\cdot}6 \leqslant L/D \leqslant 9$ (simple adjustment of distances was possible in the range of $1{\cdot}6 \leqslant L/D \leqslant 3$).

A critical flow meter was used to maintain the constant flow rate and to prevent influence of the downstream disturbances. Experiments were carried out for $Re = 13000$, 27000 and 40000.

Static pressure taps on the cylinder surface were 0·5 mm dia. Static pressure was measured in the symmetry plane using a Fortier type micromanometer with an accuracy of 1/100 mm of ethyl alcohol.

For heat-transfer experiments another measuring tube (Fig. 4) was constructed. The electric heater—4, made of Kanthal wire, uniformly wound around a Pyrex glass tube, was placed inside a thin walled metal tube—1 ($\delta = 1{\cdot}75$ mm), with a thin air layer left between. Surface temperatures were measured in the symmetry plane with an accuracy less than 0·01 mV using Cu–Ko thermocouples—3.

In order to avoid the circumferential heat conduction through the tube wall, a segment 10 mm wide (corresponding to 11° of the central angle of the tube cross section) and 200 mm long was separated with two grooves—2, filled with insulation material (epoxy resin). In this manner, constant heat flux was ensured. The heat flux on the cylinder surface was determined from the measured electric power of the heater. To avoid the error caused by end effects, the voltage drop was measured only along the length of the segment.

By rotating the tubes it was possible to place the thermocouple or static pressure tap in any desired position along the perimeter. Only one cylinder was always heated.

By measuring the pressure and temperature distributions in two planes along the cylinder (50 mm and 100 mm apart from the symmetry plane) it was checked that there was no influence of side walls on the measurements in symmetry plane.

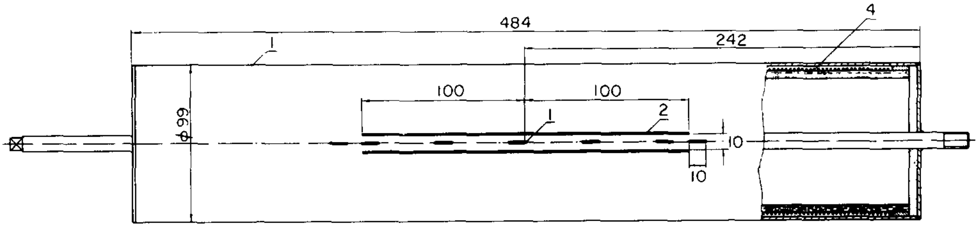

FIG. 4. Temperature measuring tube.

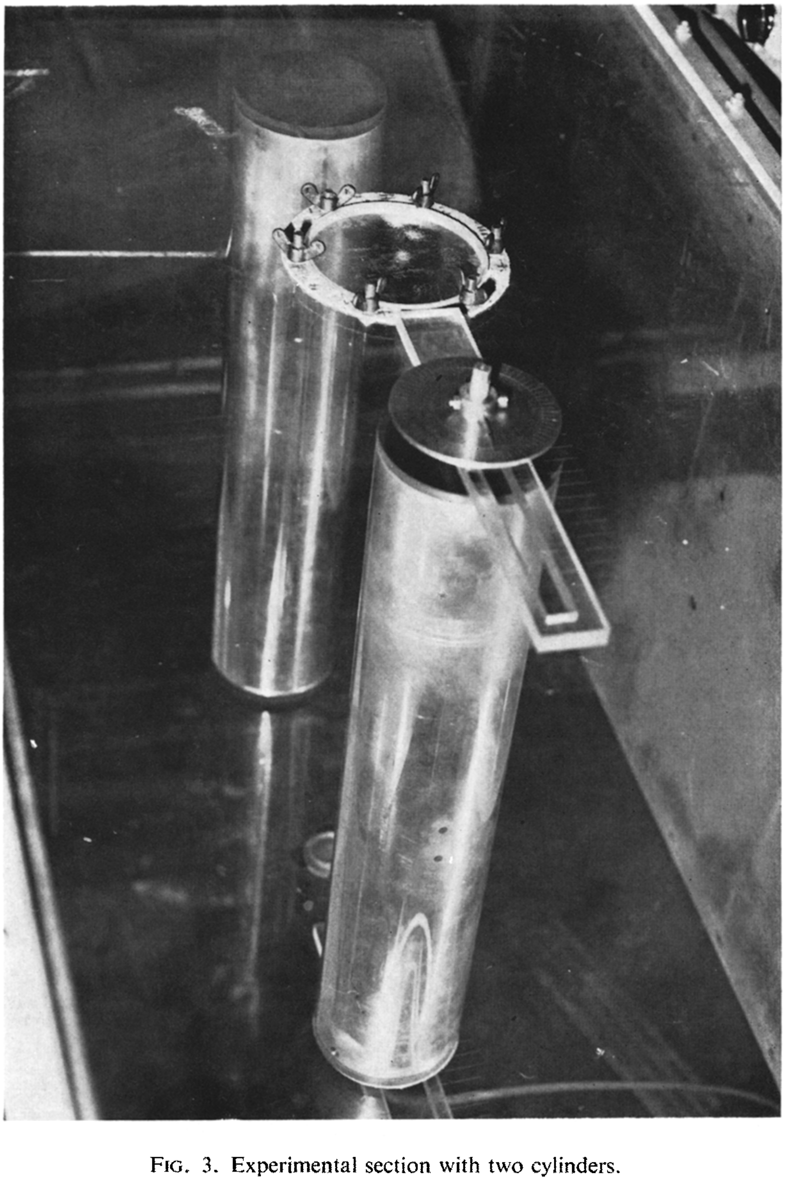

FIG. 3. Experimental section with two cylinders.

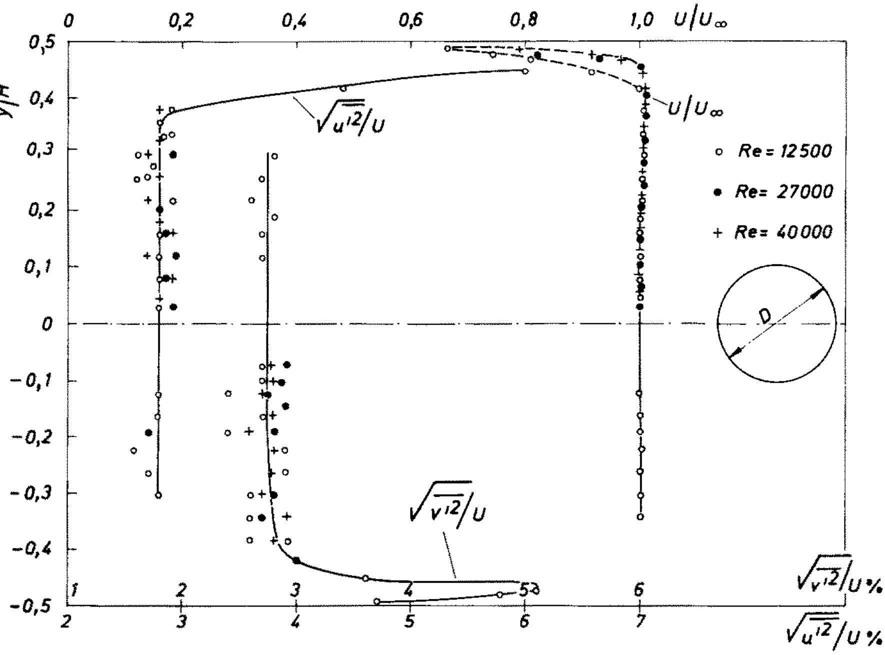

FIG. 5. Velocity and turbulence intensity profiles upstream of the cylinders.

During the experiments the flow upstream of the 1st cylinder had uniform mean velocity profile and turbulence intensity 2·8 per cent (Fig. 5).

The accuracy of the pressure and temperature measurements have been checked by comparing the results obtained for a single cylinder in cross flow with those obtained by other authors [9, 14–16]. A correction for finite blockage factor influence [14] was made in advance. Very good agreement was found for the form drag coefficient, mean Nusselt number and the local heat-transfer coefficient distribution in the laminar boundary layer.

Main stream turbulence intensity being 2·8 per cent in our experiment had no significant influence on heat transfer at front stagnation point of the single cylinder (the 1st cylinder too). The results agree very well with those for zero turbulence intensity in other experiments [14, 15, 17]. This is probably because of large cylinder diameter, and as a consequence the very small ratio of the main stream turbulence macroscale to the cylinder diameter [11].

Detailed description of the experimental apparatus, measuring tubes construction, measurement methods and the comparison of the experimental results obtained for the single cylinder with other authors results are given in [18–20].

4. EXPERIMENTAL RESULTS AND DISCUSSION

4.1 *Distribution of the local static pressure coefficient and Nusselt numbers on the surface of the cylinders*

(a) *The first cylinder.* Distribution of the local static pressure coefficient and Nusselt numbers for the 1st cylinder differ very slightly from those for the single cylinder (Figs. 6 and 7). At all distances ($\infty \geqslant L/D \geqslant 1{\cdot}6$) the laminar boundary layer develops on the surface of the 1st cylinder, up to the separation point (SL) which

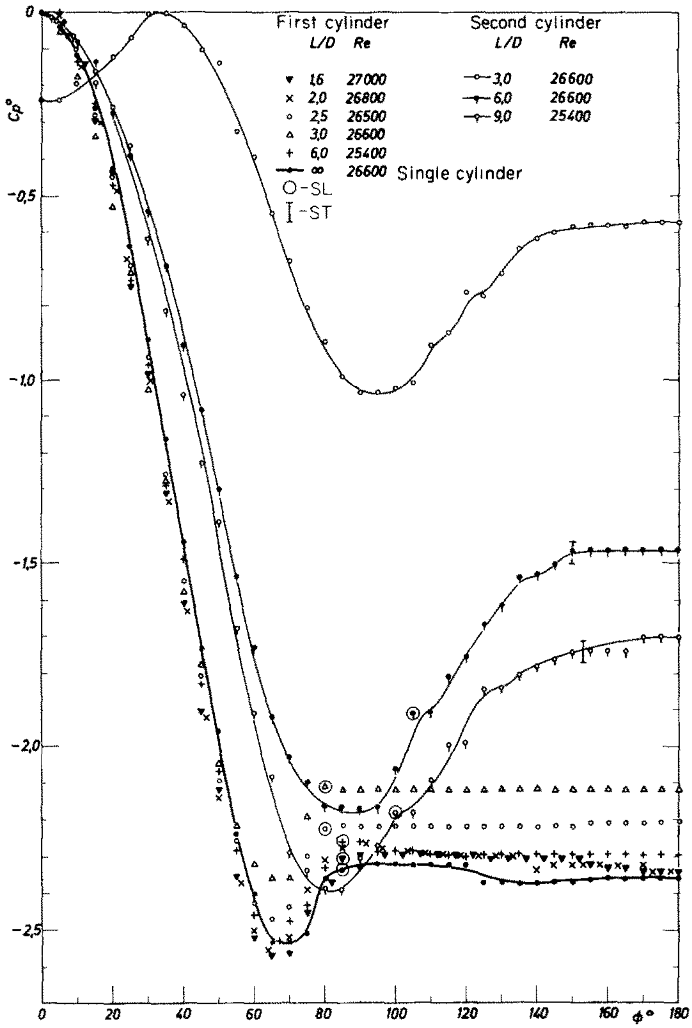

FIG. 6. First and second cylinder: perimetric distribution of the local static pressure coefficients for $Re \approx 27000$.

is between 80° and 85°.* The difference can be noticed only on the rear side at short mutual distances ($1{\cdot}6 \leqslant L/D \leqslant 3$), when a closed vortex region is formed between the cylinders. In this region local Nusselt numbers are increased for distances $L/D = 2{\cdot}5$ and 2, probably because of the more intensive and better defined vortex flow between the cylinders. For $L/D = 1{\cdot}6$ velocities in the closed vortex region evidently are smaller, so Nusselt numbers are again decreased.

(b) *The second cylinder.* Distribution of the local static pressure coefficient and Nusselt numbers for the 2nd cylinder show that, at distances $L/D = 9$ and 6 (when the closed vortex region is not yet formed between the

* Separation point (*SL*) corresponds to the beginning of the nearly constant local static pressure coefficient (Fig. 6), and the minimum Nusselt number (Fig. 7).

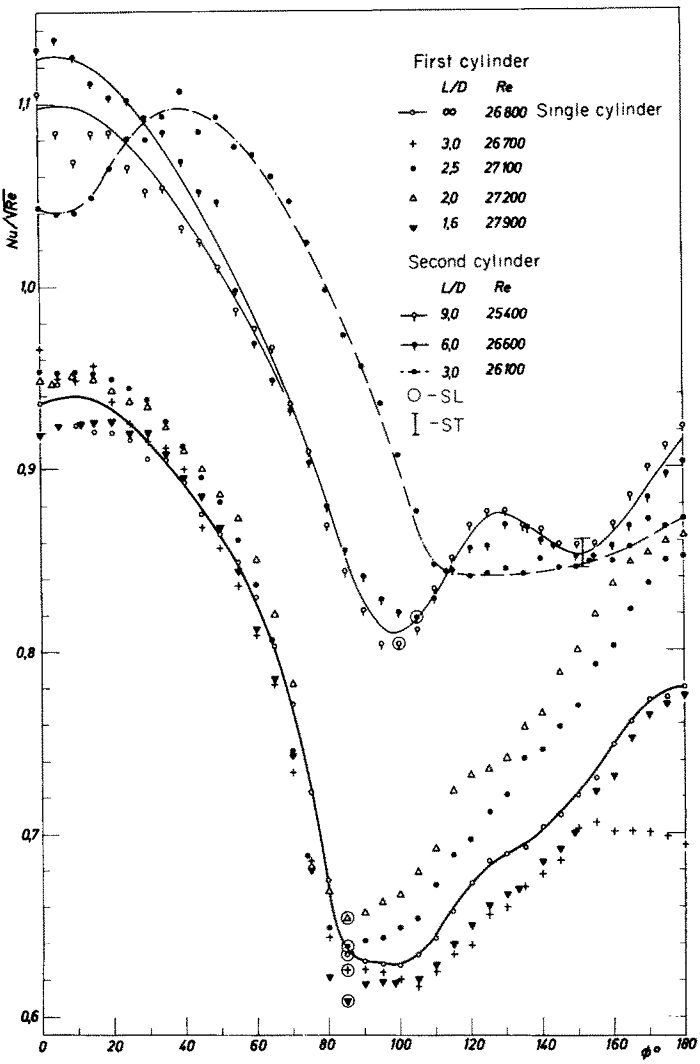

FIG. 7. First and second cylinder: perimetric distribution of the local Nusselt number for $Re \approx 27000$.

cylinders) the laminar boundary layer becomes turbulent before separation.*

Two minima on the Nusselt number distribution curves correspond to the laminar boundary-layer transition (SL) and the turbulent boundary layer separation point (ST).

* Transition point (again SL) corresponds to the inflection point on the static pressure coefficient curve (Figs. 6 and 8), and to the upstream minimum Nusselt number (Figs. 7 and 9).

The heat-transfer intensity within the laminar boundary layer is also increased by about 20 per cent (when compared to that of the 1st cylinder, Fig. 7). This is the result of the increased turbulence level ($A/D \approx 1$) of the oncoming flow.

Local Nusselt number distributions for the 2nd cylinder are practically the same at the distances $L/D = 9$ and $L/D = 6$ (Fig. 7) for the same Reynolds number. A certain difference at the front stagnation point is chiefly due to the difference in the velocity profile of the oncoming flow.

Qualitatively different local static pressure coefficient and Nusselt number distributions on the surface of the 2nd cylinder are obtained heat transfer coefficient is obtained near the reattachment points, but for $L/D > 2$ slightly downstream, and for $L/D \leqslant 2$ slightly upstream (Fig. 9).

At $Re \approx 27000$ and 40000 [19] (Figs. 8 and 9)

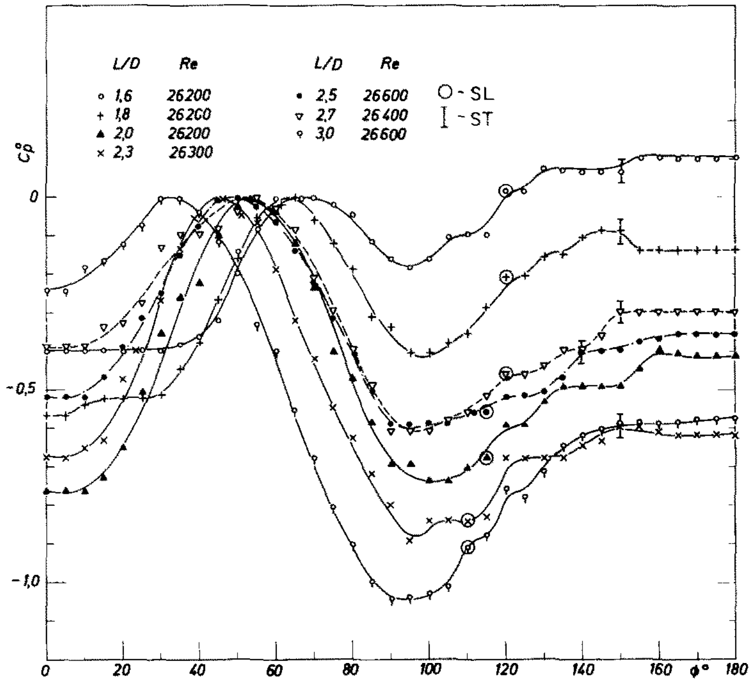

FIG. 8. Second cylinder: perimetric distribution of the local static pressure coefficients for $Re \approx 27000$.

at the distances $L/D \leqslant 3$. In this case, two reattachment points are obtained at the front side of the 2nd cylinder. They are actually the impingment points of the free shear layers separated from the 1st cylinder and which correspond to the maxima on the local static pressure coefficient distribution curves (Fig. 8). Apparently, the closed vortex region is already formed at these distances.*

Starting from these reattachment points laminar boundary layers are formed towards the front and rear stagnation points. The maximum the laminar boundary layers becomes turbulent at all mutual distances. Transition points (SL) are shifted downstream (compared with $L/D = 9$ and 6), but turbulent boundary layer separation points are approximately at the same place.

At the lowest Reynolds number (≈ 13000) only, and at some mutual distances between cylinders, transition of the laminar to turbulent boundary layer does not occur on the surface of the 2nd cylinder.

The positions of the transition point (SL) and that of the laminar (SL) or turbulent (ST) boundary-layer separation point at various mutual distances between the cylinders and for the three Reynolds numbers is shown in Table 1.

* The greatest distance at which two pressure maxima were observed on the front side of the 2nd cylinder (critical distance L_{CR}) is 3·8 D.

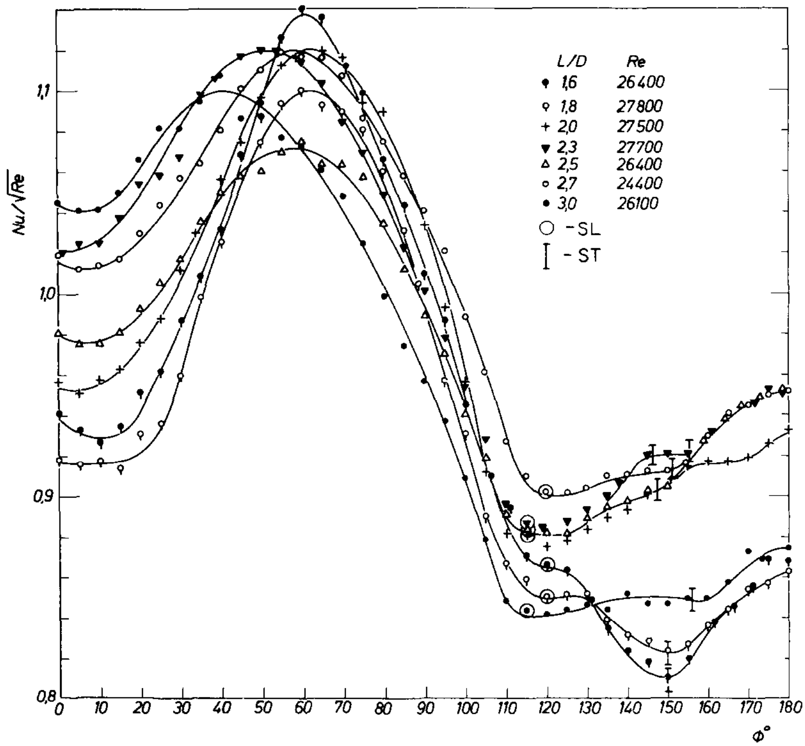

FIG. 9. Second cylinder: perimetric distribution of the local Nusselt number for $Re \approx 27000$.

Table 1.

Re = 13 000

L/D	∞	9·0	6·0	3·0	2·7	2·5	2·3	2·0	1·8	1·6
SL	85°	100°	100°	125°	125°	122°	120°	118°	120°	122°
ST	—	115°	115°	—	—	—	—	145°	145°	145°

Re = 27 7000

L/D	∞	9·0	6·0	3·0	2·7	2·5	2·3	2·0	1·8	1·6
SL	85°	98°	103°	110°	117°	112°	110°	113°	118°	118°
ST	—	148°	150°	150°	150°	143°	150°	150°	150°	150°

Re = 40 000

L/D	∞	9·0	6·0	3·0	2·7	2·5	2·3	2·0	1·8	1·6
SL	85°	95°	95°	115°	117°	115°	115°	115°	115°	115°
ST	—	150°	150°	155°	155°	150°	145°	145°	150°	150°

The front part of the 2nd cylinder is in the closed vortex region for this small distances. Due to the decrease of the velocities and the increase in length and thickness of the boundary layer developing towards the former front stagnation point, the local Nusselt numbers on the cylinder decrease simultaneously with the decrease of L/D, but are still higher than on the rear side of the 1st cylinder.

4.2 *Similarity of the local static pressure coefficient and Nusselt number distributions on the 2nd cylinder*

When expressed as the dependence of the local static pressure coefficient C_p^0 on ϕ^-, the results of the local static pressure measurements on the surface of the 2nd cylinder indicate similarity of the flow in the laminar boundary layer (downstream of the reattachment points, Fig. 10). Experimental points obtained for distances $L/D \leqslant 3{\cdot}0$, are all grouped around one curve (full line), only the results for $L/D = 1{\cdot}6$ slightly deviates. In Fig. 10 dashed line is for single cylinder and dash-dotted line for $L/D = 6$. Distribution for $L/D = 6$ is almost the same as for $L/D \leqslant 3{\cdot}0$.

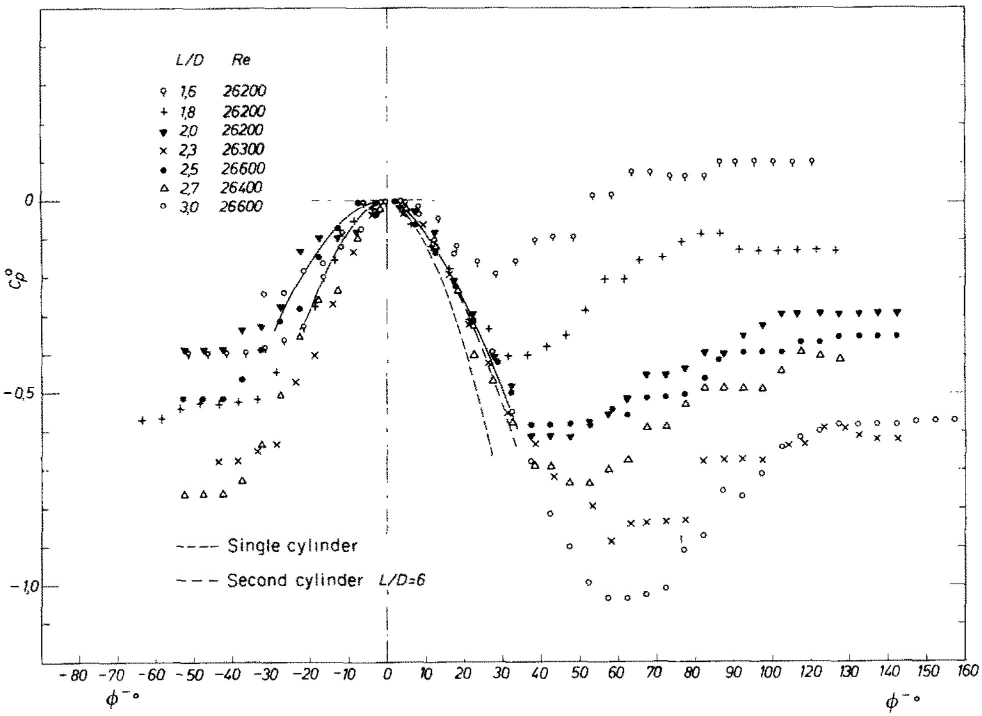

FIG. 10. Second cylinder: generalization of the results. Perimetric distribution of the local Nusselt number for $Re \approx 27000$.

No essential change in the distribution of the local static pressure coefficient and Nusselt numbers has been observed with variation of Reynolds number in the region in which measurements were performed. From this reason the plots of local C_p^0 and $Nu/\sqrt{Re}$ values are not given for all Reynolds numbers.

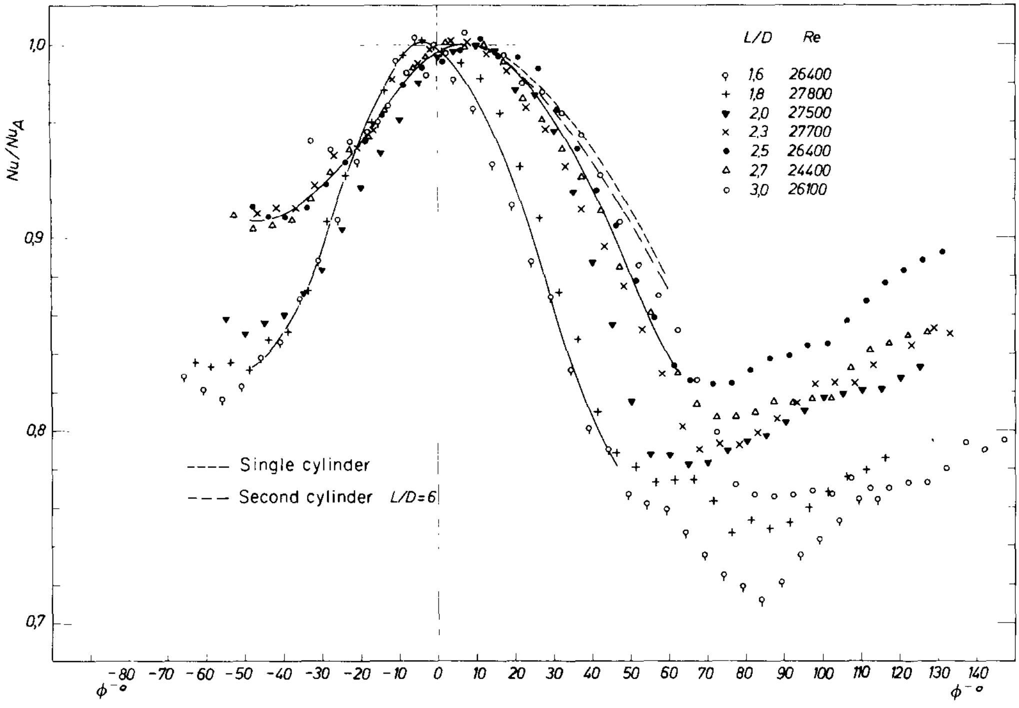

FIG. 11. Second cylinder: generalization of the results. Perimetric distribution of the local static pressure coefficients for $Re \approx 27000$.

Heat transfer similarity in the laminar boundary layer downstream of the reattachment points on the 2nd cylinder results from the similarity of flow. In Fig. 11 the heat transfer results are shown as the ratio of the local Nusselt number and maximum Nusselt number near reattachment point, again vs. ϕ^-. Obtained distributions for distances $2 < L/D \leqslant 3$ are grouped around one curve with very slight deviations (smaller than ± 2 per cent). Results for $L/D = 1{\cdot}8$ and $1{\cdot}6$ are grouped around another curve. The values for $L/D = 2$ do not belong to either of the groups (Fig. 11). Dot-dashed ($L/H = 6$) and dotted (single cylinder) lines (Fig. 11) are not very different from those for $2 < L/D \leqslant 3$, in spite of the closed vortex region being formed between two cylinders for $L/D \leqslant 3$.

Similar results are obtained for other two Reynolds numbers. This can be seen from Fig. 12, where the ratio Nu/Nu_A is presented vs. ϕ^+, for characteristic distances: $L/D = 1{\cdot}8$; $2{\cdot}3$; $2{\cdot}5$; $2{\cdot}7$ and 3, for all three Reynolds numbers. Two different lines are obtained for $L/D < 2$ and $L/D \geqslant 2{\cdot}3$, again. So, we can write:

$$Nu/Nu_A = f_1(\phi^+) \quad \text{for} \quad 6 \leqslant L/D \leqslant 9 \qquad (1)$$

$$Nu/Nu_A = f_2(\phi^+) \quad \text{for} \quad 2{\cdot}7 \geqslant L/D \geqslant 2{\cdot}3 \qquad (2)$$

$$Nu/Nu_A = f_3(\phi^+) \quad \text{for} \quad 1{\cdot}8 \geqslant L/D \geqslant 1{\cdot}6 \qquad (3)$$

$$1{\cdot}3 \,.\, 10^4 \leqslant Re \leqslant 4 \,.\, 10^4$$

Maximum Nusselt number (Nu_A) on the surface of the second cylinder depends only on the Reynolds number (Fig. 13) and, with an

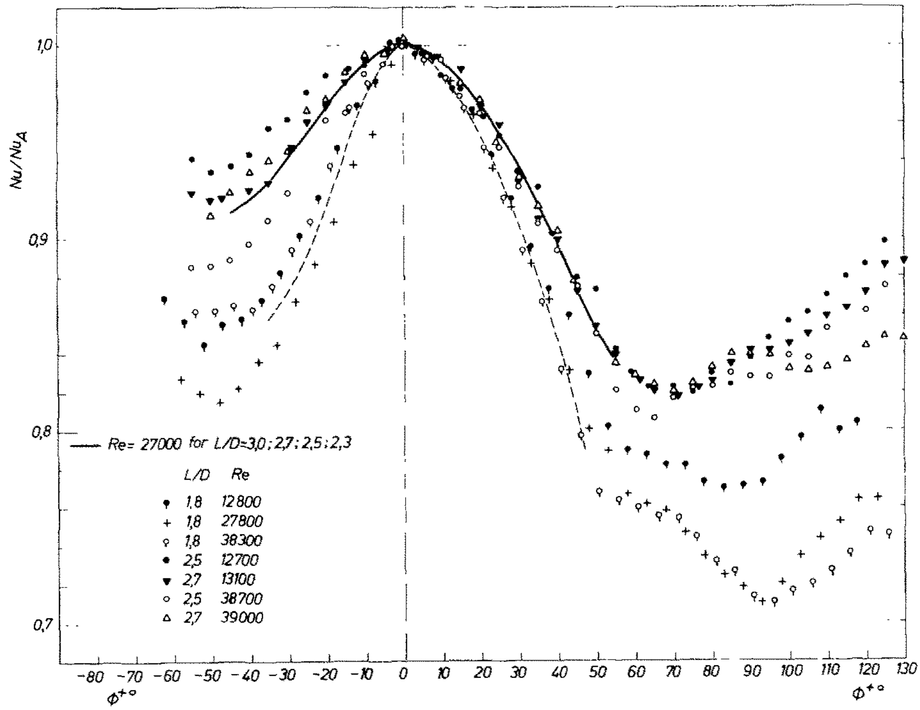

FIG. 12. Second cylinder, generalization of the results: perimetric distribution of the local Nusselt number for three Reynolds numbers.

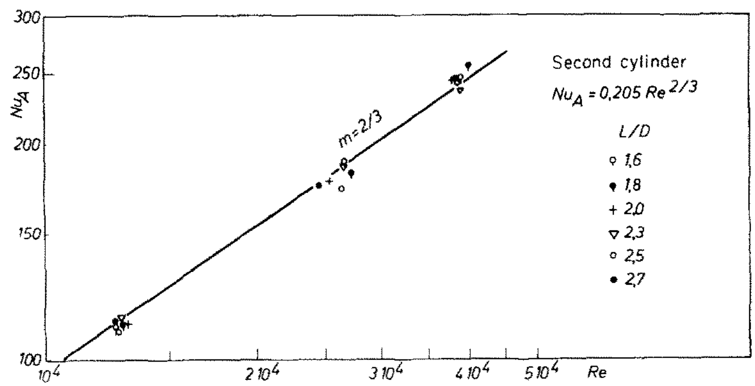

FIG. 13. Second cylinder: dependence of the maximum Nusselt number on Reynolds number, for $L/D \leqslant 2{\cdot}7$.

error smaller than ± 4 per cent can be expressed by the equation:

$$Nu_A = 0{\cdot}205\, Re^{\frac{2}{3}} \qquad (4)$$

$$1{\cdot}6 \leqslant L/D \leqslant 2{\cdot}7$$

$$1{\cdot}3\,.\,10^4 \leqslant Re \leqslant 4\,.\,10^4$$

which is in good agreement with the results of Knight [13] obtained for reattachment point in different flow conditions.

No similarity in the flow and heat transfer has been found in the vortex region on the front side of the 2nd cylinder but it might be assumed approximately that the heat transfer in this region can be expressed by one function for $2{\cdot}7 \geqslant L/D \geqslant 2{\cdot}3$ and other for $1{\cdot}8 \geqslant L/D \geqslant 1{\cdot}6$ with an error not greater than ± 4 per cent (Figs. 11 and 12).

When the closed vortex region is not yet formed between the cylinders with the same error we can write (Fig. 14):

$$Nu_A = 0{\cdot}655\, Re^{0{\cdot}55} \qquad (5)$$

$$3 \leqslant L/D \leqslant 9$$

$$1{\cdot}3\,.\,10^4 \leqslant Re \leqslant 4\,.\,10^4$$

which is the same law as for the single cylinder (Fig. 14). The results for $L/D = 3$ obeys this law, probably because the vortex region on the front side of the 2nd cylinder is still narrow.

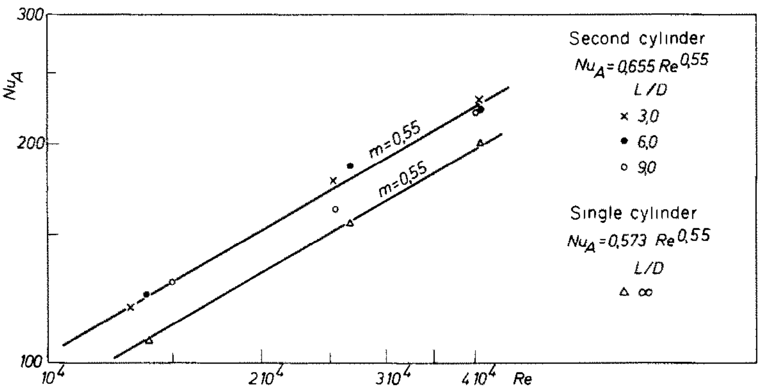

FIG. 14. Second cylinder: dependence of the maximum Nusselt number on Reynolds number, for $L/D > 2{\cdot}7$.

4.3 *Form drag coefficient and mean Nusselt number*

(a) *The first cylinder.* The 1st cylinder form drag coefficient differs slightly from that of the single cylinder (Fig. 15a, $L = \infty$). Slight changes can be noticed at shorter distances when closed vortex region is formed. Form drag is dependent on Reynolds number which is not a characteristic of single cylinder in this Reynolds number range.

The mean Nusselt number for the 1st cylinder, and the distance $L/D = 3$ does not show any difference from those for a single cylinder except at $Re = 13000$. At shorter distances deviations of the mean Nusselt number found for the 1st cylinder ($3 \geqslant L/D \geqslant 1{\cdot}6$) are no greater than ± 8 per cent as compared by those

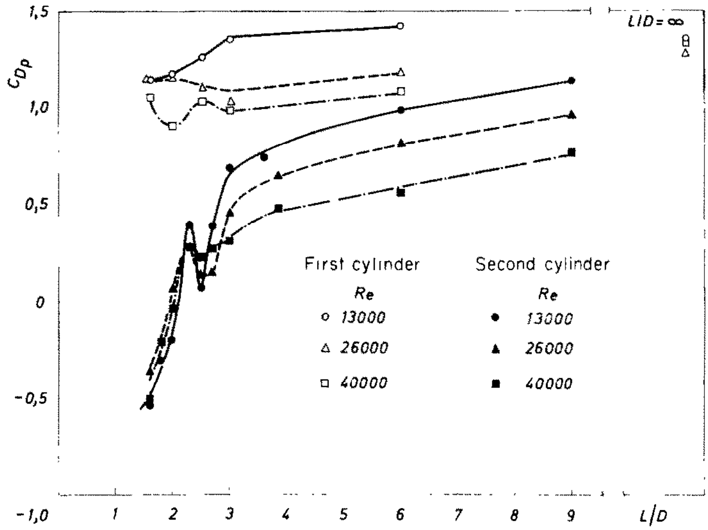

FIG. 15a. First and second cylinder: form drag coefficients in dependence on distance between cylinders.

found for the single cylinder (Fig. 16), with tendency of decreasing for $L/D < 2$.

(b) *The second cylinder.* The form drag coefficient for the 2nd cylinder is significantly smaller than that for the single (and the first) cylinder, even at large distances between cylinders ($L/D = 9$, Fig. 15a). When the closed vortex region is formed between the cylinders the form drag coefficient for the 2nd cylinder suddenly begins to decrease, and goes negative for $L/D < 2$. Certain irregularities are evident for $2{\cdot}5 \geqslant L/D \geqslant 2{\cdot}3$ (Fig. 15b). Form drag is dependent on Reynolds number, again.

For $L/D > 3{\cdot}8$ (when the closed vortex

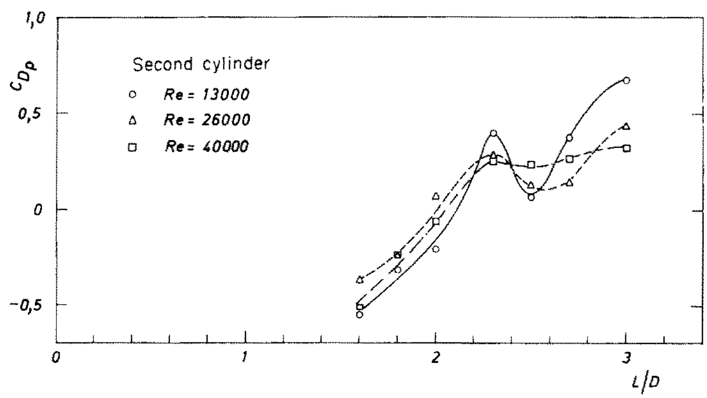

FIG. 15b. Second cylinder: form drag coefficient in dependence of the distance between cylinders, for distances less than critical.

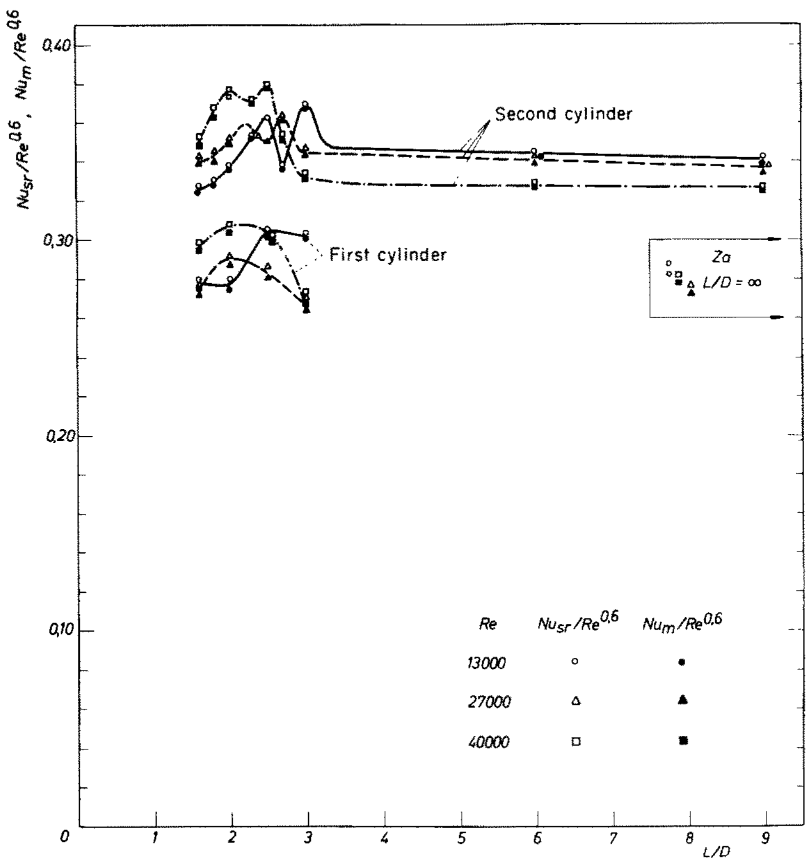

FIG. 16. First and second cylinder: dependence of the mean Nusselt number on distance between cylinders.

region is not formed) the mean Nusselt number for the 2nd cylinder is 25 per cent higher than that found for the single cylinder (Fig. 16). With the decrease of the distance between the cylinders mean Nusselt number remains constant, but only until the beginning of the closed vortex region formation (Fig. 16).

When the closed vortex region is formed between the cylinders irregularities appear in the dependence of the mean Nusselt number for distance about $L/D = 2{\cdot}3$. The minimum Nusselt number which is at $L/D = 2{\cdot}7$ for $Re = 13000$, is shifted to $L/D = 2{\cdot}3$ for $Re = 40000$ (Fig. 16).

(c) *The dependence of the mean Nusselt number on Reynolds number.* When the large part (about 50 per cent) of the cylinder surface (either of the 1st or the 2nd cylinder) is covered by the laminar boundary layer, the mean Nusselt number ($Nu_m \approx Nu_{sr}$) is proportional to $Re^{0{\cdot}6}$ (Fig. 17) as follows:

(i) the single cylinder and the 1st cylinder:

$$Nu_{sr} = Nu_m = 0{\cdot}286\, Re^{0{\cdot}6} \qquad (6)$$

$$1{\cdot}2\,.\,10^4 \leqslant Re \leqslant 4\,.\,10^4$$

$$1{\cdot}6 \leqslant L/D < \infty$$

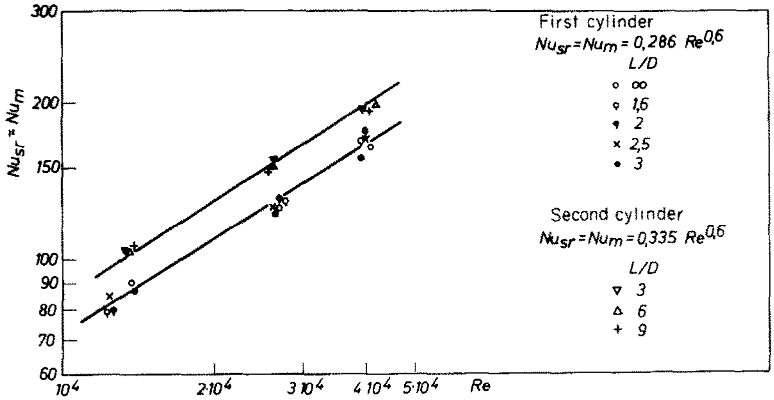

FIG. 17. First and second cylinder: mean Nusselt number in dependence on Reynolds number.

(ii) the second cylinder:

$$Nu_{sr} = Nu_m = 0{\cdot}335\,Re^{0{\cdot}6} \qquad (7)$$

$$1{\cdot}2\,.\,10^4 \leqslant Re \leqslant 4\,.\,10^4$$

$$9 \geqslant L/D \geqslant 3$$

When the largest part of the cylinder surface is in the vortex region the mean Nusselt number changes as $Re^{2/3}$. Therefore for the 2nd cylinder (Fig. 18) the mean Nusselt number expression is as follows:

$$Nu_{sr} = Nu_m = 0{\cdot}180\,Re^{2/3} \qquad (8)$$

$$1{\cdot}2\,.\,10^4 \leqslant Re \leqslant 4\,.\,10^4$$

$$1{\cdot}8 \leqslant L/D \leqslant 2{\cdot}7.$$

(d) *The comparison with other experiments.* The comparison of the total form drag of the two cylinders C_D with the results of J. R. Pannell *et al.* [2] has been shown in Fig. 19. The results of J. R. Pannell were obtained for $Re \approx 9000$, i.e. Reynolds number lower than lowest in our experiments. Qualitative and quantitative agreement is quite satisfactory.

The comparison of the mean Nusselt number for the second cylinder with those for the internal transverse rows of some in-line tube banks [1] is shown in Fig. 20. The velocity in the narrowest cross section between the tubes in the bank was chosen as the characteristic. To correspond with the tube bank results as the characteristic

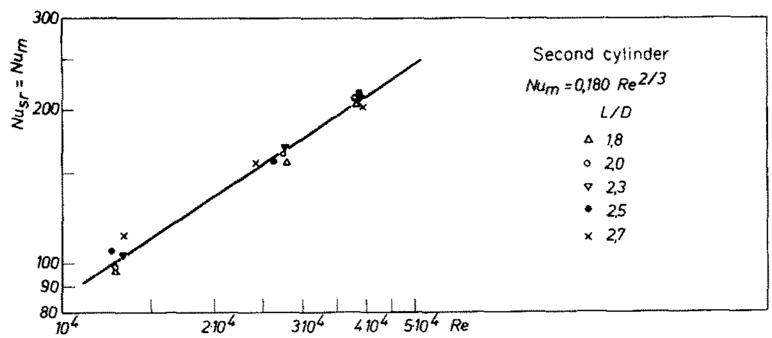

FIG. 18. Second cylinder: mean Nusselt number in dependence on Reynolds number, for $L/D \leqslant 2{\cdot}7$.

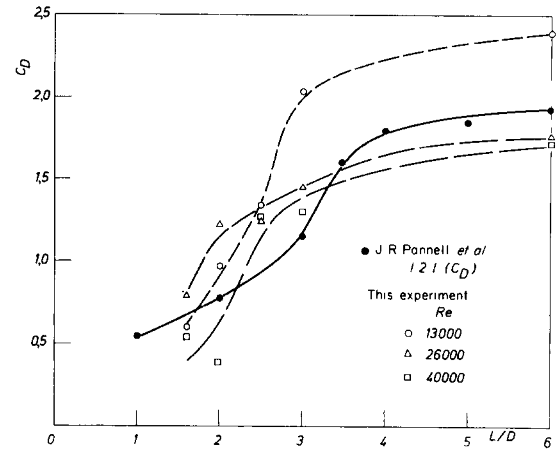

FIG. 19. Comparison of the form drag coefficient for two cylinders with results of J. R. Pannell.

velocity for the 2nd cylinder was chosen the velocity in the channel cross section where the separation occurs on the first cylinder. Quite good agreement was obtained.

5. THE FLOW PATTERN

Knowing the local static pressure coefficient and Nusselt number distributions on the 1st and 2nd cylinder, positions of the laminar or turbulent boundary layer separation points on the both cylinders (Table 1) and mechanism of the flow around single cylinder it was possible to develop an idea of the pattern and mechanism of the flow around the two cylinders. The sketches of the two different characteristic flow patterns are shown on Fig. 21.

The laminar boundary layer on the surface of the 1st cylinder separates (SL) at 80–85° from the front stagnation point for all values of L/D.

At distances between cylinders greater than certain critical distance (L_{CR}), when the closed vortex region is not formed between them, the second cylinder has no influence on the processes developing on the first one. It can then be concluded that the pattern and mechanism

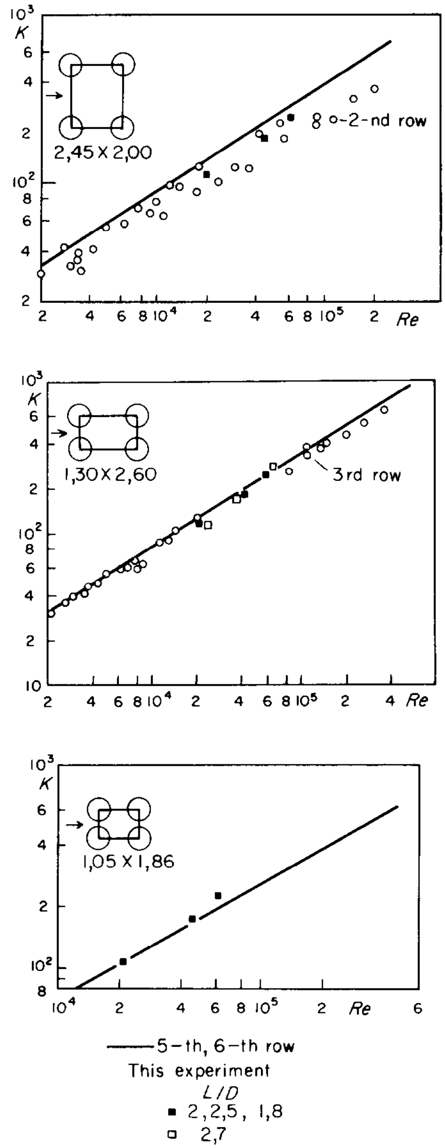

FIG. 20. Second cylinder: comparison of the results for mean Nusselt number with the results of A. A. Zhukauskas.

of flow in the wake of the 1st cylinder is the same as that for the single cylinder.

As the 2nd cylinder is in the wake of the 1st the increased turbulence level of the oncoming flow causes the transition (SL) of the laminar to turbulent boundary layer to occur before

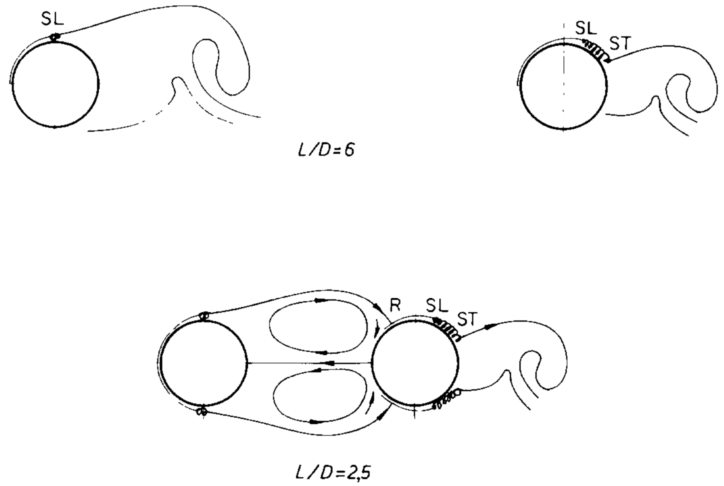

FIG. 21. Two different flow patterns around two cylinders.

the separation (ST). The wake of the 2nd cylinder is thus more narrow which causes significant decrease of the form drag coefficient. The flow around 2nd cylinder is similar to supercritical flow ($Re > 2$–$5 \, . \, 10^5$) around single cylinder. Heat transfer in laminar boundary layer is increased due to the high turbulence intensity and macroscale of the order of cylinder diameter.

At the distances shorter than critical the free shear layers which separate from the 1st cylinder impinge on the front side of the 2nd cylinder, and the closed vortex region is formed between the cylinders. It has been found that $(L/D)_{CR}$ =

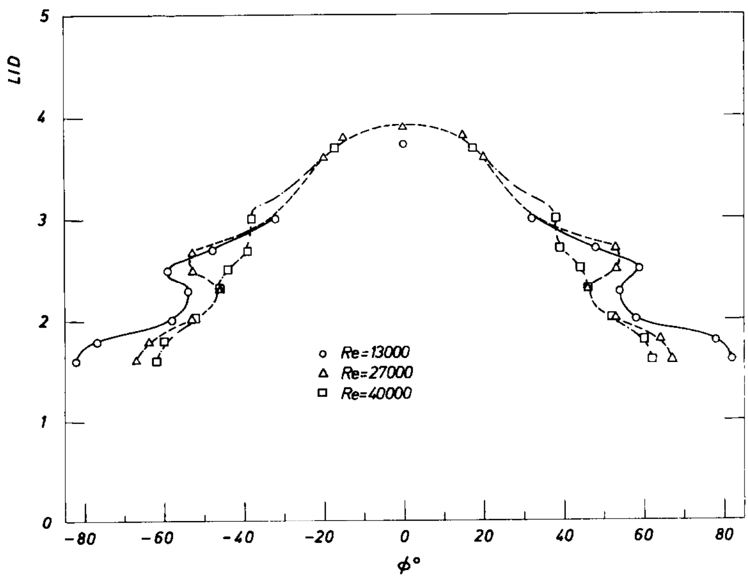

FIG. 22. Second cylinder: position of the reattachment points for different distances between cylinders.

3·8 (Fig. 22) corresponds to the critical distance at which a closed vortex region is formed. This distance is somewhat shorter than that found for the formation of the closed vortex region between the wedge and the cylinder at supersonic flow ($L_{CR}/D = 5$ for $Re = 1{\cdot}6\text{–}2\,.\,10^5$, $Ma = 1{\cdot}86\text{–}2{\cdot}78$ [3].

The shifting of the free shear layer reattachment points with the change of L/D is shown in Fig. 22. Except for the irregularity found near $L/D = 2{\cdot}3$ the width of the vortex region at the front of the second cylinder is increased when the distance decreases from the critical value to $L/D = 1{\cdot}6$. For greater Reynolds number reattachment points are nearer to the former front stagnation point. So it seems that the critical distance is greater for greater Reynolds number.

On the 2nd cylinder downstream of the reattachment point (R), a laminar boundary layer is formed which either separates or becomes turbulent before separation (SL). For all values of L/D (Table 1) at which the laminar boundary layers becomes turbulent before flow separation (with the exception of the distances $L/D = 9$ and 6 for $Re = 13000$), the width of the vortex region at the rear side of the 2nd cylinder does not essentially change with the change of distance.

When the distance between two cylinders is smaller than the length of the vortex formation region, the vortices behind the 1st cylinder are formed in the limited space and we can expect the change of the flow character around the 2nd cylinder. The length of the vortex formation region of the single cylinder for the Reynolds number range used in our experiment is $2{\cdot}4 \geqslant L/D \geqslant 1{\cdot}4$, according to the results obtained by Bloor [5]. Also, a characteristic discontinuity in the frequency of vortex shedding behind the cylinder and the plate (Fig. 23) was obtained at the distance $l/D = 1{\cdot}6$ [4]. (According to the Fig. 23, $l \approx L - D$ if the cylinder is in place of the plate.)

There are numerous indications, in our experiment, that the 2nd cylinder enters into the vortex formation region of the 1st cylinder when the distance between them is $L/D = 2{\cdot}3$; these are as follows:

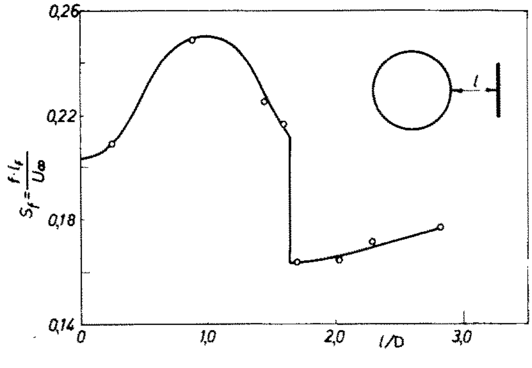

FIG. 23. Frequency of shedding behind cylinder and flat plate.

(i) It has been noticed that the similarity of the flow (Fig. 10) and heat transfer (Figs. 11 and 12) on the front side of the 2nd cylinder exists approximately for two groups of distances $2{\cdot}7 \geqslant L/D \geqslant 2{\cdot}3$ and $1{\cdot}8 \geqslant L/D \geqslant 1{\cdot}6$.

(ii) Certain characteristic irregularities of the form drag coefficient (Fig. 15) and the mean Nusselt number (Fig. 16) for the 2nd cylinder have been observed at the distance $L/D \approx 2{\cdot}3$.

(iii) On the curve (Fig. 22) indicating the relationship between the reattachment point position on the 2nd cylinder and the distance between cylinders a characteristic irregularity was obtained for $L/D \approx 2{\cdot}3$.

(iv) The transition to the turbulent boundary layer on the 2nd cylinder occurs only at $L/D \leqslant 2{\cdot}3$ for $Re = 13000$. For higher Reynolds numbers the transition to turbulence on the 2nd cylinder occurs for the greater distances also, so entrance of the 2nd cylinder into the vortex formation region of the 1st (at $L/D = 2{\cdot}3$) does not change the character of flow.

6. CONCLUSIONS

The most important results can be summarized as follows:

— although the measurements were performed only on the surface of the cylinders, the results obtained enable us to draw conclusions about the character and pattern of the flow around two cylinders,

— in the range in which our measurements were performed, the Reynolds number does not essentially influence the mechanism and pattern of the flow around two cylinders,

— at the mutual distance $L/D = 3{\cdot}8$ (critical distance) a closed vortex region (or free cavity) is formed between cylinders. It should be expected that at distances shorter than the critical one, the flow within the closed vortex region becomes quasi stationary, especially when the second cylinder has entered into the vortex formation region of the first. The second cylinder begins to enter into vortex formation region at $L/D \approx 2{\cdot}3$,

— the similarity of the flow and heat transfer in the laminar boundary layer downstream of the reattachment points on the second cylinder has been found,

— the Nusselt number at the reattachment points on the second cylinder depend only on Reynolds number. For $L/D > 2{\cdot}7$ is $Nu_A \sim Re^{0{\cdot}55}$ as for flow over the single cylinder. For $L/D \leqslant 2{\cdot}7$ is $Nu_A \sim Re^{\frac{2}{3}}$, as for reattachment points in other flow conditions,

— the dependence of mean Nusselt number on Reynolds number also differs depending on whether the distance is greater or smaller than $2{\cdot}7\ D$. For $L/D > 2{\cdot}7$ is $Nu_m \sim Re^{0{\cdot}6}$ for the 1st and 2nd cylinder, as for a single cylinder in subcritical regime. For $L/D \leqslant 2{\cdot}7$ is $Nu_m \sim Re^{\frac{2}{3}}$ for the 2nd cylinder, as for vortex regions,

— investigation of the flow field between and around two cylinders should be undertaken using hot-wire anemometer, visualization methods, or some more suitable experimental methods as laser-Doppler method, to prove more exactly conclusions drawn here about flow pattern.

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ECOULEMENT DE FLUIDE ET TRANSFERT THERMIQUE POUR DEUX CYLINDRES EN ATTAQUE FRONTALE

Résumé—On a mené une recherche expérimentale sur l'écoulement frontal et le transfert thermique autour de deux cylindres l'un étant dans le sillage de l'autre. Par des mesures on a déterminé les distributions des coefficients de pression statique et le nombre de Nusselt sur les surfaces des cylindres. Les résultats expérimentaux et la connaissance des comportements de l'écoulement autour du cylindre unique rend difficile la formulation d'une idée sur le mécanisme et le type de l'écoulement autour des deux cylindres, et aussi d'expliquer l'influence mutuelle des cylindres et les changements caractéristiques des coefficients de trainée et de transfert thermique moyen.

On a fait varier le nombre de Reynolds de $1{,}2 \cdot 10^4$ à 4.10^4 et la distance entre les cylindres, telle que $1{,}6 \geqslant L/D \geqslant 9$.

Deux configurations caractéristiques différentes d'écoulement ont été trouvées suivant que la distance entre les cylindres est supérieure ou inférieure à 3,8 D.

Pour $L < 3{,}8\ D$ se forme une région tourbillonnaire fermée entre les cylindres.

Le nombre moyen de Nusselt pour le premier cylindre est comme pour le cylindre unique proportionnel à $Re^{0{,}6}$ pour toute distance L.

Le nombre de Nusselt pour le second cylindre est proportionnel à $Re^{0{,}6}$ pour $L > 2{,}7\ D$ et à $Re^{\frac{2}{3}}$ pour $L \geqslant 2{,}7\ D$.

STRÖMUNGSVERHALTEN UND WÄRMEÜBERTRAGUNG BEI ZWEI QUER ANGESTRÖMTEN ZYLINDERN

Zusammenfassung—Eine experimentelle Untersuchung der Strömungsverhältnisse und des Wärmeübergangs bei zwei querangeströmten Zylindern (die Zylinder waren hintereinander angeordnet) wurde durchgeführt. Die Verteilung des statischen Druckkoeffizienten und der Nusselt-Zahl über der Oberfläche der beiden Zylinder wurde gemessen. Unsere experimentellen Ergebnisse und die Kenntnis der Strömungsverhältnisse um einen Zylinder erlauben uns ein Modell für die Strömung zwischen und um zwei Zylinder zu entwickeln, um damit den gegenseitigen Einfluss der Zylinder und der charakteristischen Grössen auf den Formwiderstand und den mittleren Wärmeübergangskoeffizienten zu erklären.

Die Reynolds-Zahl im Bereich $1\text{-}2 \cdot 10^4 \leqslant Re \leqslant 4 \cdot 10^4$ und der Abstand zwischen den beiden Zylindern im Bereich $1.6 \leqslant L/D \leqslant 9$ variiert.

Für die beiden Fälle, Abstand zwischen den beiden Zylindern kleiner oder grösser $3.8 \cdot D$, wurden zwei unterschiedliche charakteristische Strömungsformen festgestellt. Für $L < 3.8 \cdot D$ wurde ein geschlossener Wirbelbereich zwischen den Zylindern beobachtet.

Die mittlere Nusselt-Zahl für den ersten Zylinder ist proportional $Re^{0{,}6}$, so wie für den Fall eines einzelnen Zylinders.

Die mittlere Nusselt-Zahl für den zweiten Zylinder ist im Bereich $L > 2.7 \cdot D$ proportional $Re^{0{,}6}$ und proportional $Re^{\frac{2}{3}}$ im Bereich $L \leqslant 2.7 \cdot D$.

ТЕЧЕНИЕ ЖИДКОСТИ И ПЕРЕНОС ТЕПЛА ПРИ ПОПЕРЕЧНОМ ОБТЕКАНИИ ДВУХ ЦИЛИНДРОВ

Аннотация—Выполнено экспериментальное исследование потока и переноса тепла вокруг двух поперечно—обтекаемых цилиндров (один в следе другого). Эамерены распределения коэффициентов статического давления и числа Нуссельта на поверхностях обоих цилиндров. Наши экспериментальные результаты и знание процессов, происходящих вокруг обтекаемого единичного цилиндра, позволяют нам составить представление о механизме и картине течения между двумя цилиндрами и вокруг них, а также объяснить взаимное влияние цилиндров и характерные изменения в сопротивлении и среднем коэффициенте теплообмена.

Значения числа Рейнольдса изменялись в диапазоне $1{,}2 \times 10^4 \leqslant Re \leqslant 4 \times 10^4$, а расстояние между цилиндрами в интервале $1{,}6 \leqslant L/D \leqslant 9$.

Были обнаружены две различные характерные картины течения в зависимости от того, больше или меньше $3{,}8D$ расстояние между цилиндрами. При $L < 3{,}8D$ между цилиндрами образуется замкнутая вихревая область.

Среднее число Нуссельта для 1го цилиндра пропорционально $Re^{0{,}6}$, как для единичного цилиндра при всех расстояниях.

Среднее число Нуссельта для 2го цилиндра пропорционально $Re^{0{,}6}$, при $L < 2{,}7\ D$ и $Re^{2/3}$ при $L \leqslant 2{,}7D$.